$$\Rightarrow a = \frac{\Sigma Y}{n}$$

And, $$\Sigma XY = a\,(0) + b\,\Sigma X^2$$

$$\Rightarrow b = \frac{\Sigma XY}{\Sigma X^2}$$

5. By substituting the obtained values of 'a' and 'b' in equation (i), we get the trend line of best fit.

Example 11

Given below are the consumer price index numbers (CPI) of the industrial workers.

Year	2014	2015	2016	2017	2018	2019	2020
Index Number	145	140	150	190	200	220	230

Find the best fitted trend line by the method of least squares and tabulate the trend values.

Solution

Note that the number of years is Odd

$$\Rightarrow n = odd$$

Procedure:

1. Take middle year value as A i.e. $A = 2017$
2. Find $X = x_i - A$
3. Find X^2 and XY

Year (x_i)	Index number (Y)	$X = x_i - A$ $= x_i - 2017$	X^2	XY	Trend value $Y_t = a + bX$
2014	145	-3	9	-435	152.1+ (-3)×16.6 = 102.3
2015	140	-2	4	-280	152.1+ (-2)×16.6 = 118.9
2016	150	-1	1	-150	152.1+ (-1)×16.6 = 135.5
2017	190	0	0	0	152.1+ (0)×16.6 = 152.1
2018	200	1	1	200	152.1+ (1)×16.6 = 168.7
2019	220	2	4	440	152.1+ (2)×16.6 = 185.3
2020	230	3	9	690	152.1+ (3)×16.6 = 201.9
n = 7	**$\Sigma Y = 1065$**	**$\Sigma X = 0$**	**$\Sigma X^2 = 28$**	**$\Sigma XY = 465$**	**$\Sigma Y_t = 1064.7$**

$$a = \frac{\Sigma Y}{n} = \frac{1065}{7} = 152.14$$

and $$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{465}{28} = 16.6$$

Therefore, the required equation of the straight-line trend is given by

$$y = a + bx \Rightarrow \mathbf{y = 152.1 + 16.6x}$$

Example 12

Based on the data available for the sales of an item in a district, by the method of least squares

(*i*) tabulate the trend values

(*ii*) find the best fit for a straight-line trend

(*iii*) compute expected sale trend for year 2002

Year	1996	1997	1998	1999	2000	2001
Sales (In lakh ?)	6.5	5.3	4.3	6.1	5.6	7.8

Note that the number of years is even

$$\Rightarrow n = \text{even}$$

Procedure:

1. Take middle year value as i.e. $= \dfrac{\text{sum of two middle years}}{2} = \dfrac{1998+1999}{2} = 1998.5$

2. Find $X = \dfrac{x_i - A}{0.5}$ (we divide by 0.5 to avoid cumbersome calculations)

3. Find X^2 and XY

Solution:

Year (x_i)	Index number (Y)	$X=\dfrac{x_i-A}{0.5}=\dfrac{x_i-1998.5}{0.5}$	X^2	XY	Trend value $Y_t = a + bX$
1996	6.5	-5	25	-32.5	5.9+ (-5) × 0.13 = 5.25
1997	5.3	-3	9	-15.9	5.9+ (-3) × 0.13 = 5.51
1998	4.3	-1	1	-4.3	5.9+ (-1) × 0.13 = 5.77
1999	6.1	1	1	6.1	5.9+ (1) × 0.13 = 6.03
2000	5.6	3	9	16.8	5.9+ (3) × 0.13 = 6.29
2001	7.8	5	25	39	5.9+ (5) × 0.13 = 6.55
n = 6	**ΣY = 35.6**	**ΣX = 0**	**ΣX² = 70**	**ΣXY = 9.2**	**ΣY$_t$ = 35.4**

$$a = \frac{\Sigma Y}{n} = \frac{35.6}{6} = 5.9$$

and $$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{9.2}{70} = 0.13$$

Therefore, the required equation of the straight-line trend is given by

$$y = a + bx \Rightarrow \mathbf{y = 5.9 + 0.13x}$$

According to the line trend, the predicted sales for year 2002:

$$y = 5.9 + 0.13\left(\frac{x_i - A}{0.5}\right) = 5.9 + 0.13\left(\frac{2002-1998.5}{0.5}\right) = \text{₹ } \mathbf{6.81 \text{ lakh}}$$

Note: *1. Future trend forecast made by using this method are based only on the trend values*

2. The predicted trend values by using this method are more reliable than any other method for year 2006, $x = -1$

$\therefore \quad y_t = 138.86 + 7.64 \times (-1) = 131.22$

for year 2007, $x = 0$

$\therefore \quad y_t = 138.86 + 7.64 \times (0) = 138.86$

for year 2008, $x = 1$

$\therefore \quad y_t = 138.86 + 7.64 \times 1 = 146.50$

for year 2009, $x = 2$

$\therefore \quad y_t = 138.86 + 7.64 \times 2 = 154.14$

for year 2010, $x = 3$

$\therefore \quad y_t = 138.86 + 7.64 \times 3 = 161.78$

Example 2. *Fit a straight line trend by the method of least squares and tabulate the trend values from the following data:*

Year	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*
Sales (in ₹ crores)	*6.7*	*5.3*	*4.3*	*6.1*	*5.6*	*7.9*	*5.8*	*6.1*

Solution. Here, $n = 8$ (even)

So, origin is mean of two middle years

i.e. $\dfrac{1998 + 1999}{2} = 1998.5$

Construct the table as under:

Year t	*Sales* y	$x = t_i - 1998.5$	x^2	xy	*Trend values* $y_t = a + bx$
1995	6.7	−3.5	12.25	−23.45	5.62
1996	5.3	−2.5	6.25	−13.25	5.72
1997	4.3	−1.5	2.25	−6.45	5.82
1998	6.1	−0.5	0.25	−3.05	5.92
1999	5.6	0.5	0.25	2.8	6.03
2000	7.9	1.5	2.25	11.85	6.13
2001	5.8	2.5	6.25	14.5	6.23
2002	6.1	3.5	12.25	21.35	6.33
	$\Sigma y = 47.8$	$\Sigma x = 0$	$\Sigma x^2 = 42$	$\Sigma xy = 4.3$	

$$a = \frac{\Sigma y}{n} = \frac{47.8}{8} \Rightarrow a = 5.975$$

and $b = \dfrac{\Sigma xy}{\Sigma x^2} = \dfrac{4.3}{42} \Rightarrow b = 0.102$

So, the required equation of the straight line trend is

$$y_t = 5.975 + 0.102x$$

So, trend values

for year 1995, $x = -3.5$

$\therefore \quad y_t = 5.975 + 0.102 \times (-3.5) = 5.62$